Introduction

• Chapter 3 deals with flexure
• Beams must have adequate safety margin against other types of failure
• They may be more dangerous—more uncertain and more catastrophic
• Shear failure is one such failure
• It is not fully understood and sudden without warning
• Special shear reinforcement are provided to ensure flexural failure would occur before shear failure if overloading happens.
Shear analysis and design are not really concerned about shear as such.

The real concern is DIAGONAL TENSION
FIGURE 4.1
Shear failure of reinforced concrete beam: (a) overall view, (b) detail near right support.
Diagonal Tension in homogeneous beams

(a)

(b)

(c)

(d) $v_{av} = \frac{V}{ab}$

$V_{max} = \frac{3}{2} v_{av}$
FIGURE 4.3
Stress trajectories in homogeneous rectangular beam.

Principal Stresses
compressive stresses and a pair of inclined tensile stresses that act at right angles to each other. They are known as *principal stresses* (Fig. 4.3e). Their value, as mentioned in Section 3.2, is given by

\[ t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + \nu^2} \]  \hspace{1cm} (3.1)

and their inclination \( \alpha \) by \( \tan 2\alpha = 2\nu/f \).
RC BEAMS WITHOUT SHEAR REINFORCEMENTS
RC Beams without shear reinforcements

• No flexural failure unlike plain concrete beam
• Diagonal tension failure occurs elsewhere
• Caused by shear alone or combined action of shear and flexure
FIGURE 4.4
Typical locations of critical combinations of shear and moment.
Web-shear crack

- when flexure small
- Near neutral axis
- Shear stress equals tensile strength

\[ \nu_{cr} = \frac{V_{cr}}{bd} = 3.5 \sqrt{f'_{c}} \] (4.2a)
Flexure shear cracking

- Both moment and shear significant
- Fails at a lower stress due to preexisting cracks due to flexure

\[ \nu_{cr} = \frac{V_{cr}}{bd} = 1.9 \sqrt{f'_c} \]
• The shear at which diagonal cracks develop depends on the ratio of shear force to bending moment

• more precisely on the $v$ to $f$ near the top of flexural crack

$$v_{cr} = \frac{V_{cr}}{bd} = 1.9\sqrt{f'_c} + 2500 \frac{\rho V_d}{M} \leq 3.5\sqrt{f'_c}$$ (4.3a)

where

$$V_{cr} = v_{cr} bd$$
FIGURE 4.6
Correlation of Eq. (4.3a) with test results.
Behaviour of diagonally cracked beam

- Flexural cracks are harmless as longitudinal steel is present
- If shear reinforcement is not present, diagonal cracks is critical and determines the strength
Two types of behaviour as diagonal cracks formed:

- The diagonal cracks spread immediately from tension face to compression, splitting it into two and failing - for shallow beams (span to depth 8 or more)

- For deeper beams, the crack spreads partially into compression zone. Failure load is significantly higher
Once crack is formed

\[ V_{\text{int}} = V_{cz} + V_d + V_{iy} \]

Dowel V is small and can cause splitting
Formation of diagonal crack produce following redistribution of stress

- Shear stress increases on the uncracked area
- Compression force increases
- Tension in steel increases

- Failure occurs in various ways-yielding, crushing, splitting, pull out
- Relatively deep beam can take significant load after formation of diagonal crack, but this reserve strength is erratic and is ignored
RC BEAM WITH WEB REINFORCEMENT
Types of web reinforcement

(a) Vertical stirrups

(b) Stirrup support bars
(c) Main reinforcing bars

(d) Bent-up longitudinal bars
formation. After diagonal cracks have developed, web reinforcement augments the shear resistance of a beam in four separate ways:

1. Part of the shear force is resisted by the bars that traverse a particular crack. The mechanism of this added resistance is discussed below.
2. The presence of these same bars restricts the growth of diagonal cracks and reduces their penetration into the compression zone. This leaves more uncracked concrete available at the head of the crack for resisting the combined action of shear and compression, already discussed.
3. The stirrups also counteract the widening of the cracks, so that the two crack faces stay in close contact. This makes for a significant and reliable interface force $V_i$ (see Fig. 4.7).
4. As shown in Fig. 4.8, the stirrups are arranged so that they tie the longitudinal reinforcement into the main bulk of the concrete. This provides some measure of restraint against the splitting of concrete along the longitudinal reinforcement, shown in Figs. 4.1 and 4.7b, and increases the share of the shear force resisted by dowel action.

From this it is clear that failure will be imminent when the stirrups start yielding. This not only exhausts their own resistance but also permits a wider crack opening with consequent reduction of the beneficial restraining effects, points 2 to 4, above.
Beams with vertical stirrups

**FIGURE 4.9**
Forces at a diagonal crack in a beam with vertical stirrups.

\[ V_{\text{ext}} = V_{cz} + V_d + V_{iy} + V_s \]  \hspace{1cm} (a)

\[ V_c = V_{cz} + V_d + V_{iy} \]  \hspace{1cm} (b)
The number of stirrups \( n \) spaced a distance \( s \) apart was seen to depend on the length \( p \) of the horizontal projection of the diagonal crack. This length is conservatively assumed to be equal to the effective depth of the beam; thus \( n = d/s \), implying a crack somewhat flatter than 45°. Then, at failure, when \( V_{\text{ext}} = V_n \), Eqs. (a) and (b) yield for the nominal shear strength

\[
V_n = V_c + \frac{A_v f_{yt} d}{s} \quad (4.7a)
\]

where \( V_c \) is taken equal to the cracking shear \( V_{cr} \) given by Eq. (4.3a); that is,

\[
V_c = \left( 1.9 \sqrt{f'_c} + 2500 \frac{\rho V_d}{M} \right) bd \leq 3.5 \sqrt{f'_c} bd \quad (4.3a)
\]

Dividing both sides of Eq. (4.7a) by \( bd \), the same relation is expressed in terms of the nominal shear stress:

\[
\nu_n = \frac{V_n}{bd} = \nu_c + \frac{A_v f_{yt}}{bs} \quad (4.7b)
\]
Beams with inclined bars

\[ V_n = V_c + \frac{A_v f_{yf} d (\sin \alpha + \cos \alpha)}{s} \]  

(4.9)
According to ACI Code 11.1.1, the design of beams for shear is to be based on the relation

\[
V_u \leq \phi V_n
\]  

where \( V_u \) is the total shear force applied at a given section of the beam due to factored loads and \( V_n = V_c + V_s \) is the nominal shear strength, equal to the sum of the contributions of the concrete and the web steel if present. Thus for vertical stirrups

\[
V_u \leq \phi V_c + \frac{\phi A_y f_y d}{s}
\]  

(4.11a)

and for inclined bars

\[
V_u \leq \phi V_c + \frac{\phi A_y f_y d (\sin \alpha + \cos \alpha)}{s}
\]  

(4.11b)

where all terms are as previously defined. The strength reduction factor \( \phi \) is to be taken equal to 0.75 for shear. The additional conservatism, compared with the value of \( \phi = 0.90 \) for bending for typical beam designs, reflects both the sudden nature of diagonal tension failure and the large scatter of test results.
EXAMPLE 4.1 Beam without web reinforcement. A rectangular beam is to be designed to carry a shear force $V_u$ of 27 kips. No web reinforcement is to be used, and $f'_c$ is 4000 psi. What is the minimum cross section if controlled by shear?

SOLUTION. If no web reinforcement is to be used, the cross-sectional dimensions must be selected so that the applied shear $V_u$ is no larger than one-half the design shear strength $\phi V_c$. The calculations will be based on Eq. (4.12b). Thus,

$$V_u = \frac{1}{2} \phi (2\lambda \sqrt{f'_c b_w d})$$

$$b_w d = \frac{27,000}{0.75 \times 1.0 \sqrt{4000}} = 569 \text{ in}^2$$

A beam with $b_w = 18$ in. and $d = 32$ in. is required. Alternately, if the minimum amount of web reinforcement given by Eq. (4.13) is used, the concrete shear resistance may be taken at its full value $\phi V_c$, and it is easily confirmed that a beam with $b_w = 12$ in. and $d = 24$ in. will be sufficient.
FIGURE 4.12
Location of critical section for shear design: (a) end-supported beam; (b) beam supported by columns; (c) concentrated load within $d$ of the face of the support; (d) member loaded near the bottom; (e) beam supported by girder of similar depth; (f) beam supported by monolithic vertical element.
\[ V_c = \left( 1.9 \lambda \sqrt{f'_c} + 2500 \frac{\rho_w V_u d}{M_u} \right) b_w d \leq 3.5 \lambda \sqrt{f'_c} b_w d \] (4.12a)

where \( \rho_w \) = longitudinal reinforcement ratio \( A_s / b_w d \) or \( A_s / bd \). With the section dimension \( b_w \) and \( d \) in inches and \( V_u d \) and \( M_u \) in consistent units, \( V_c \) is expressed in pounds. In Eq. (4.12a), the quantity \( V_u d / M_u \) is not to be taken greater than 1.0.

\[ V_c = 2 \lambda \sqrt{f'_c} b_w d \] (4.12b)

- Conservative where shear-moment ratio is high
The term $\lambda$ in Eq. (4.12a) is a modification factor reflecting the lower tensile strength of lightweight concrete compared with normalweight concrete of the same compressive strength (see Table 2.2 and Ref. 4.13). Lightweight aggregate concretes having densities from 90 to 120 pcf are used widely, particularly for precast elements. In accordance with ACI Code 8.6.1, $\lambda = 0.85$ for “sand-lightweight” concrete and 0.75 for “all-lightweight” concrete. Linear interpolation between 0.75 and 0.85, based on volumetric fractions, is permitted when a portion of the lightweight fine aggregate is replaced by normalweight fine aggregate. Linear interpolation between 0.85 and 1.0 is also permitted for concretes containing normalweight fine aggregate and a blend of lightweight and normalweight coarse aggregate. If the average split-cylinder strength of lightweight concrete (a good measure of its direct tensile strength) is specified, $\lambda = f_{ct} / (6.7 \sqrt{f_{ct}'} ) \leq 1.0$. For normalweight concrete, $\lambda = 1.0$.

The tests on which Eqs. (4.12a) and (4.12b) are based used beams with concrete compressive strength mostly in the range of 3000 to 5000 psi. More recent experimental results (Refs. 4.14 to 4.17) have shown that in beams constructed using high-strength concrete (see Section 2.12) with $f_{ct}'$ above 6000 psi, the concrete contribution to shear strength $V_c$ is less than predicted by those equations. Differences become increasingly significant, the higher the concrete strength. For this reason, ACI Code 11.1.2 places an upper limit of 100 psi on the value of $\sqrt{f_{ct}'}$ to be used in Eqs. (4.12a) and (4.12b), as well as in all other ACI Code shear provisions. However, values of $\sqrt{f_{ct}'}$ greater than 100 psi may be used in computing $V_c$ if a minimum amount of web reinforcement is used (see Section 4.5b).
Minimum web reinforcement

If \( V_u \), the shear force at factored loads, is no larger than \( \phi V_c \), calculated by Eq. (4.12a) or alternatively by Eq. (4.12b), then theoretically no web reinforcement is required. Even in such a case, however, ACI Code 11.4.6 requires provision of at least a minimum area of web reinforcement equal to

\[
A_{v,\text{min}} = 0.75 \sqrt{\frac{f'_c}{f_{yt}}} \frac{b_w s}{f_{yt}} \geq 50 \frac{b_w s}{f_{yt}}
\]  

(4.13)

where \( s \) = longitudinal spacing of web reinforcement, in.

\( f_{yt} \) = yield strength of web steel, psi

\( A_{v,\text{min}} \) = total cross-sectional area of web steel within distance \( s \), in\(^2\)

This provision holds unless \( V_u \) is one-half or less of the design shear strength provided by the concrete \( \phi V_c \). Specific exceptions to this requirement for minimum
c. Region in Which Web Reinforcement Is Required

If the required shear strength \( V_u \) is greater than the design shear strength \( \phi V_c \) provided by the concrete in any portion of a beam, there is a theoretical requirement for web reinforcement. Elsewhere in the span, web steel at least equal to the amount given by Eq. (4.13) must be provided, unless the factored shear force is less than \( \frac{1}{2} \phi V_c \).

The portion of any span through which web reinforcement is theoretically necessary can be found from the shear diagram for the span, superimposing a plot of the shear strength of the concrete. Where the shear force \( V_u \) exceeds \( \phi V_c \), shear reinforcement must provide for the excess. The additional length through which at least the minimum web steel is needed can be found by superimposing a plot of \( \phi V_c / 2 \).
EXAMPLE 4.2

Limits of web reinforcement. A simply supported rectangular beam 16 in. wide having an effective depth of 22 in. carries a total factored load of 9.4 kips/ft on a 20 ft clear span. It is reinforced with 7.62 in$^2$ of tensile steel, which continues uninterrupted into the supports. If $f' = 4000$ psi, throughout what part of the beam is web reinforcement required?

SOLUTION. The maximum external shear force occurs at the ends of the span, where $V_u = 9.4 \times 20/2 = 94$ kips. At the critical section for shear, a distance $d$ from the support, $V_u = 94 - 9.4 \times 1.83 = 76.8$ kips. The shear force varies linearly to zero at midspan. The variation of $V_u$ is shown in Fig. 4.13a. Adopting Eq. (4.12b) gives

$$V_c = 2\lambda \sqrt{f'_c b_w d} = 2 \times 1.0 \sqrt{4000} \times 16 \times 22 = 44,500 \text{ lb}$$

Hence $\phi V_c = 0.75 \times 44.5 = 33.4$ kips. This value is superimposed on the shear diagram, and, from geometry, the point at which web reinforcement theoretically is no longer required is

$$10 \left( \frac{94.0 - 33.4}{94.0} \right) = 6.45 \text{ ft}$$

from the support face. However, according to the ACI Code, at least a minimum amount of web reinforcement is required wherever the shear force exceeds $\phi V_c/2$, or 16.7 kips in this case. As seen from Fig. 4.13a, this applies to a distance

$$10 \left( \frac{94.0 - 16.7}{94.0} \right) = 8.22 \text{ ft}$$
from the support face. To summarize, at least the minimum web steel must be provided within a distance of 8.22 ft from the supports, and within 6.45 ft the web steel must provide for the shear force corresponding to the shaded area.

If the alternative Eq. (4.12a) is used, the variation along the span of \( \rho_w \), \( V_u \), and \( M_u \) must be known so that \( V_c \) can be calculated. This is shown in tabular form in Table 4.1.

The factored shear \( V_u \) and the design shear capacity \( \phi V_c \) are plotted in Fig. 4.13b. From the graph it is found that stirrups are theoretically no longer required 6.39 ft from the support face. However, from the plot of \( \phi V_c / 2 \) it is found that at least the minimum web steel is to be provided within a distance of 8.26 ft.

When Figs. 4.13a and b are compared, it is evident that the length over which web reinforcement is needed is nearly the same for this example whether Eq. (4.12a) or (4.12b) is used. However, the smaller shaded area of Fig. 4.13b indicates that substantially less web-steel area would be needed within that required distance if the more accurate Eq. (4.12a) were adopted.
**TABLE 4.1**  
Shear design example

<table>
<thead>
<tr>
<th>Distance from Support, ft</th>
<th>$M_{u'}$ ft-kips</th>
<th>$V_{u'}$ kips</th>
<th>$V_c^a$</th>
<th>$\phi V_c$</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>94.0</td>
<td>61.3</td>
<td>46.0</td>
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<td>89</td>
<td>84.6</td>
<td>61.3</td>
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<td>57.8</td>
<td>43.4</td>
</tr>
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<td>38.9</td>
</tr>
<tr>
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<td>301</td>
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<td>48.8</td>
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</tr>
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</tr>
<tr>
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<td>45.6</td>
<td>34.2</td>
</tr>
<tr>
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<td>428</td>
<td>28.2</td>
<td>44.6</td>
<td>33.5</td>
</tr>
<tr>
<td>8</td>
<td>451</td>
<td>18.8</td>
<td>43.8</td>
<td>32.8</td>
</tr>
<tr>
<td>9</td>
<td>465</td>
<td>9.4</td>
<td>43.0</td>
<td>32.3</td>
</tr>
<tr>
<td>10</td>
<td>470</td>
<td>0</td>
<td>42.3</td>
<td>31.7</td>
</tr>
</tbody>
</table>

$^a V_c = (1.9 \lambda \sqrt{f_c} + 2500 \rho_w V_u d/M_u) b_w d \leq 3.5 \lambda \sqrt{f_c} b_w d$ and $V_u d/M_u \leq 1.0$
Design of web reinforcement

The design of web reinforcement, under the provisions of the ACI Code, is based on Eq. (4.11a) for vertical stirrups and Eq. (4.11b) for inclined stirrups or bent bars. In design, it is usually convenient to select a trial web-steel area $A_v$, based on standard stirrup sizes [usually in the range from No. 3 to 5 (No. 10 to 16) for stirrups, and according to the longitudinal bar size for bent-up bars], for which the required spacing $s$ can be found. Equating the design strength $\phi V_n$ to the required strength $V_u$ and transposing Eqs. (4.11a) and (4.11b) accordingly, one finds that the required spacing of web reinforcement is, for vertical stirrups,

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} \quad (4.14a)$$

and for bent bars

$$s = \frac{\phi A_v f_{yt} d (\sin \alpha + \cos \alpha)}{V_u - \phi V_c} \quad (4.14b)$$
Where web reinforcement is needed, the Code requires it to be spaced so that every $45^\circ$ line, representing a potential diagonal crack and extending from the middepth $d/2$ of the member to the longitudinal tension bars, is crossed by at least one line of web reinforcement; in addition, the Code specifies a maximum spacing of 24 in. When $V_s$ exceeds $4 \sqrt{f'_c b_w d}$, these maximum spacings are halved. These limitations are shown in Fig. 4.14 for both vertical stirrups and inclined bars, for situations in which the excess shear does not exceed the stated limit.

For design purposes, Eq. (4.13) giving the minimum web-steel area $A_v$ is more conveniently inverted to permit calculation of maximum spacing $s$ for the selected $A_v$. Thus, for the usual case of vertical stirrups, with $V_s \leq 4 \sqrt{f'_c b_w d}$, the maximum spacing of stirrups is the smallest of

$$s_{\text{max}} = \frac{A_v f_{yl}}{0.75 \sqrt{f'_c b_w}} \leq \frac{A_v f_{yl}}{50 b_w}$$  \hspace{1cm} (4.15a)

$$s_{\text{max}} = \frac{d}{2}$$  \hspace{1cm} (4.15b)

$$s_{\text{max}} = 24 \text{ in.}$$  \hspace{1cm} (4.15c)

For longitudinal bars bent at $45^\circ$, Eq. (4.15b) is replaced by $s_{\text{max}} = 3d/4$, as confirmed by Fig. 4.14.

To avoid excessive crack width in beam webs, the ACI Code limits the yield strength of the reinforcement to $f_{yr} = 60,000$ psi or less for reinforcing bars and 80,000 psi or less for welded wire reinforcement. In no case, according to the ACI Code, is $V_s$ to exceed $8 \sqrt{f'_c b_w d}$, regardless of the amount of web steel used.
FIGURE 4.14
Maximum spacing of web reinforcement as governed by diagonal crack interception.
EXAMPLE 4.3

Design of web reinforcement. Using vertical U stirrups with \( f_{yt} = 60,000 \) psi, design the web reinforcement for the beam in Example 4.2.

SOLUTION. The solution will be based on the shear diagram in Fig. 4.13a. The stirrups must be designed to resist that part of the shear shown shaded. With No. 3 (No. 10) stirrups used for trial, the three maximum spacing criteria are first applied. For \( \phi V_s = V_u - \phi V_c = 43,400 \) lb, which is less than \( 4\phi \sqrt{f'_c b_w d} = 66,800 \) lb, the maximum spacing must exceed neither \( d/2 = 11 \) in. nor 24 in. Also, from Eq. (4.15a),

\[
\begin{align*}
    s_{\text{max}} &= \frac{A_v f_{yt}}{0.75 \sqrt{f'_c b_w}} = \frac{0.22 \times 60,000}{0.75 \sqrt{4000 \times 16}} = 17.4 \text{ in.} \\
    &
\end{align*}
\]

\[
\begin{align*}
    \leq \frac{A_v f_{yt}}{50b_w} &= \frac{0.22 \times 60,000}{50 \times 16} = 16.5 \text{ in.} \\
\end{align*}
\]

The first criterion controls in this case, and a maximum spacing of 11 in. is imposed. From the support to a distance \( d \) from the support, the excess shear \( V_u - \phi V_c \) is 43,400 lb. In this region, the required spacing is

\[
\begin{align*}
    s &= \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60,000 \times 22}{43,400} = 5.0 \text{ in.} \\
\end{align*}
\]
This is neither so small that placement problems would result nor so large that maximum spacing criteria would control, and the choice of No. 3 (No. 10) stirrups is confirmed. Solving Eq. (4.14a) for the excess shear at which the maximum spacing can be used gives

\[ V_u - \phi V_c = \frac{\phi A_v f_{yt} d}{s} = \frac{0.75 \times 0.22 \times 60,000 \times 22}{11} = 19,800 \text{ lb} \]

With reference to Fig. 4.13a, this is attained at a distance \( x_1 \) from the point of zero excess shear, where \( x_1 = 6.45 \times 19,800/60,600 = 2.10 \text{ ft} \). This is 4.35 ft from the support face. With this information, a satisfactory spacing pattern can be selected. The first stirrup is usually placed at a distance \( s/2 \) from the support. The following spacing pattern is satisfactory:

1 space at 2 in. = 2 in.
7 spaces at 5 in. = 35 in.
2 spaces at 7 in. = 14 in.
4 spaces at 11 in. = 44 in.

Total = 95 in. = 7 ft 11 in.
Graphically

The resulting stirrup pattern is shown in Fig. 4.13c. As an alternative solution, it is possible to plot a curve showing required spacing as a function of distance from the support. Once the required spacing at some reference section, say at the support, is determined,

\[ s_0 = \frac{0.75 \times 0.22 \times 60,000 \times 22}{94,000 - 33,400} = 3.59 \text{ in.} \]

it is easy to obtain the required spacings elsewhere. In Eq. (4.14a), only \( V_u - \phi V_c \) changes with distance from the support. For uniform load, this quantity is a linear function of distance from the point of zero excess shear, 6.45 ft from the support face. Hence, at 1 ft intervals,

\[ s_1 = 3.59 \times 6.45/5.45 = 4.25 \text{ in.} \]
\[ s_2 = 3.59 \times 6.45/4.45 = 5.20 \text{ in.} \]
\[ s_3 = 3.59 \times 6.45/3.45 = 6.70 \text{ in.} \]
\[ s_4 = 3.59 \times 6.45/2.45 = 9.45 \text{ in.} \]
\[ s_5 = 3.59 \times 6.45/1.45 = 15.97 \text{ in.} \]

This is plotted in Fig. 4.15 together with the maximum spacing of 11 in., and a practical spacing pattern is selected. The spacing at a distance \( d \) from the support face is selected as the minimum
requirement, in accordance with the ACI Code. The pattern of No. 3 (No. 10) U-shaped stirrups selected (shown on the graph) is identical with the previous solution. In most cases, the experienced designer would find it unnecessary actually to plot the spacing diagram of Fig. 4.15 and would select a spacing pattern directly after calculating the required spacing at intervals along the beam.
More rigorous formula

If the web steel were to be designed on the basis of the excess-shear diagram in Fig. 4.13b, the second approach illustrated above would necessarily be selected, and spacings would be calculated at intervals along the span. In this particular case, a spacing of 7.07 in. is calculated up to 20 in. from the face of the support. The calculated spacing drops to 6.76 in. at $d$ from the face of the support, and then increases to 11 in., the maximum permissible spacing, 4 ft from the support. The following practical spacing could be used:

1 space at 3 in. = 3 in.
6 spaces at 7 in. = 42 in.
4 spaces at 11 in. = 44 in.
Total = 89 in. = 7 ft 5 in.

Thus, 11 No. 3 (No. 10) stirrups would be used, rather than the 14 previously calculated, in each half of the span.
The number of stirrups just calculated represents the minimum for each of the two expressions for $V_c$. Although not required by the ACI Code, it is good design practice to continue the stirrups (at maximum spacing) through the middle region of the beam, even though the calculated shear is low. Doing so satisfies the dual purposes of providing continuing support for the top longitudinal reinforcement that is required wherever stirrups are used and providing additional shear capacity in the region to handle load cases not considered in developing the shear diagram. If this were done, the number of stirrups would increase from 14 and 11 to $16\frac{1}{2}$ and $13\frac{1}{2}$ per half-span (i.e., one stirrup at midspan), respectively.
Torsion

- RC beams are subjected to flexure and shear
- For RC columns, Axial load is present along with bending and shear
- Torsion/twisting is sometimes present.
Figure 1 Reinforced concrete members subjected to torsion: (a) spandrel beam, (b & c) Load acting away from plane of bending (d & e) curved and circular beams.
Primary torsion

• Equilibrium or statically determinate torsion.

• No alternate load path, must be supported by torsion to maintain static equilibrium.
Secondary torsion

- Compatibility or Statically indeterminate torsion
- Arises from compatibility of deformation of adjacent parts
- If the spandrel beam has little torsional stiffness or inadequate reinforcement, torsional cracking will reduce the torsional stiffness further.
FIGURE 8.1
Torsional effects in reinforced concrete:
(a) primary or equilibrium torsion at a cantilevered slab; (b) secondary or compatibility torsion at an edge beam; (c) slab moments if edge beam is stiff torsionally; and (d) slab moments if edge beam is flexible torsionally.
FIGURE 8.2
Curved continuous beam bridge, Las Vegas, Nevada, designed for torsional effects.
(Courtesy of Portland Cement Association.)
FIGURE 8.3
Stresses caused by torsion.

(a)

(b) $\tau_{\text{max}}$

sneak stress

(c) Torsional stress

(d) Principal stresses

(e) Crack
FIGURE 8.5
Reinforced concrete beam in torsion: (a) torsional reinforcement and (b) torsional cracks.